Presentations of rings with a chain of semidualizing modules

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Let R be a commutative noetherian local ring. A finitely generated R–module C is called semidualizing if the natural homothety map $\chi_C^R : R \longrightarrow \mathrm{Hom}_R(C, C)$ is an isomorphism and $\mathrm{Ext}_R^{>0}(C, C) = 0$. Examples of semidualizing R–modules include R itself and a dualizing R–module when one exists. The set of all isomorphism classes of semidualizing R–modules is denoted by $\mathfrak{G}_0(R)$, and the isomorphism class of a semidualizing R–module C is denoted $[C]$. Each semidualizing R–module C gives rise to a notion of reflexivity for finite R–modules. For instance, each finite projective R–module is totally C–reflexive. For semidualizing R–modules C and B, we write $[C] \trianglelefteq [B]$ whenever B is totally C–reflexive. A chain in $\mathfrak{G}_0(R)$ is a sequence $[C_n] \trianglelefteq \cdots \trianglelefteq [C_1] \trianglelefteq [C_0]$, and such a chain has length n if $[C_i] \neq [C_j]$ whenever $i \neq j$.

In this talk, we are going to investigate a Cohen-Macaulay ring R which admits a dualizing module and a suitable chain in $\mathfrak{G}_0(R)$. First we show that, when a Cohen–macaulay ring R with dualizing module has a suitable chain in $\mathfrak{G}_0(R)$ of length n, then there exist a Gorenstein ring Q and ideals $I_1, \cdots, I_n$ of Q such that $R \cong Q/(I_1 + \cdots + I_n)$ and, for each $\Lambda \subseteq \{1, \cdots, n\}$, the ring $Q/(\Sigma_{l\in\Lambda} I_l)$ has certain homological and cohomological properties. Then we prove the converse of this result.